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The effect of work-hardening upon the hardness of solids: minimum hardness

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It is suggested that the hardness of many solids may approach a minimum value which the material can attain through glide processes. This occurs when the effective yield stress of the material is very low or the dislocation glide mobility very high. The minimum value of the hardness is a result of the work-hardening characteristics of the indented material. For many materials, such as f c c metals, NaCl-type alkali halides or diamond cubic solids, the minimum hardness may be calculated by use of a simple model and its magnitude is shown to agree well with that expected from the rate of second stage work-hardening in these solids.

1. Introduction

For a perfectly plastic–rigid solid the theory of hardness indentation is relatively simple. An important conclusion of the theory is that the hardness, or the average pressure under an indentor, is a constant factor of the yield stress [1]. When a Knoop or Vickers diamond indentor is used, this factor, known as the Tabor relation, is very close to three. This number has been routinely used in practice for many years to characterize engineering materials. It is only under very special circumstances, however, that real materials deform in a manner which can be approximated by a perfectly plastic–rigid solid. Therefore, it is not unexpected that measurements of hardness: yield stress ratios have often been found to vary widely.

Upon indenting such materials as perspex or nylon, for example, one discovers that the ratio of Vickers hardness to yield stress is much less than that of the Tabor relation [2]. The deviation of these solids from perfectly plastic–rigid behaviour, however, has been previously discussed and is well understood. The ratio of yield stress to elastic modulus in perspex and nylon is very high. Under an indentor, the portions of the displacement attributable to plastic and elastic deformation become comparable and the morphology, or mode of deformation under the indentor changes from that expected for a perfectly plastic–rigid solid to one reminiscent of radial expansion. Johnson [3] has shown that the average indentation pressure can then be predicted by a comparison of the indentation process to the expansion by internal pressure of a void in an infinite, elastic–plastic medium.

A more important type of discrepancy between observed hardness values and the Tabor relation, however, is found if one measures say the Vickers hardness of a high purity, well annealed metal, or a good quality single crystal. For example annealing a fully work-hardened f c c metal such as copper may reduce the room temperature hardness to a value of the order of 10^{-2} times the shear modulus; whereas the yield stress of the solid may be much lower than 10^{-4} times the shear modulus so that the hardness is a hundred times greater than the yield stress. Similar behaviour is also found for rock salt. Almost twenty years ago Westbrook [4] pointed out that for many of these single crystals the ratio of hardness to yield stress could be as high as 35. More recently, careful measurements by Chin *et al.* [5] of both hardness and yield stress for several alkali halide single crystals in which the yield stress was controlled by solute additions have shown that as the yield stress is reduced, the hardness, in fact, approaches a constant value.

In both of these examples, the deviation of the behaviour of the material from the Tabor relation is the result of the appreciable work-hardening which occurs during the initial stages of plastic deformation under the indentor. The fact that the work-hardening influences the measured hardness value of a well annealed, high purity f c c metal has been suggested, for example, by Gilman [6], but the form of this influence and its applicability to a wide variety of solids have not previously been understood. Attempts to incorporate work-hardening into a slip-line analysis or an elastic–plastic type analysis using finite element methods have been made but generally prove to be inflexible. The empirical analysis of Atkins and Tabor [7] in which a representative strain is assigned to a particular shaped indentor, has been successfully used for many years to characterize work-hardenable materials but such approaches do not lead to a fundamental understanding of the problem. It is shown below that in many limiting cases the effect of work-hardening upon the indentation process can be explained in very simple terms and that in these cases the hardness provides basic information concerning the work-hardening characteristics of such solids.

2. The minimum hardness

In many instances the stress–strain curve of a plastically deforming solid takes on a very simple form. For a high purity, well annealed solid at a temperature below which an appreciable amount of diffusional creep would be expected, one finds that over a large interval of strain, an approximately linear relationship between true stress and true strain exists. This behaviour can easily occur over the 10 or 20% true strain which is thought to be the maximum found under a Vickers indentor. The slope of the true stress, true strain relation under these conditions is the rate of hardening during stage two deformation,

$$\theta_{\text{II}} = G/A.$$

(The strain under an indentor is very complicated and involves considerable shear therefore even for a single crystal one would expect an absence of appreciable stage one deformation.) If the yield stress of the material is very small and we consider the material to deform in a plastically isotropic manner, we can then obtain an approximation to the plastic indentation problem by replacing the elastic constants of the known elastic solution with the linear plastic constant, θ_{II}. The elastic solutions to both the wedge and cone indentation problem are identical and with the use of the constant A, the average indentation pressure, H, in the plastic problem is given by

$$H = (2\tan\beta)G/A \qquad (1)$$

where G is the shear modulus, A the hardening constant, β the angle between the indentor and the indented surface, and the effective Poissons ratio is taken as one-half.

The value of the hardness predicted above represents a time independent minimum hardness which a wolid approaches when the yield stress is very small. The hardness can, in fact, be less than this minimum time independent value but diffusional creep (stage three deformation) would be required and the hardness would become time dependent. If the indentation process is controlled not by the minimum stress required for long range dislocation motion (the yield stress) but by the dynamic process of the growth of a plastic zone around an indentation, as is the case when dislocation mobility is very low, the hardness is again time dependent; in this instance it is of a greater value than the minimum hardness, even though the effective yield stress is very small.

3. Discussion

The dimensionless factor A in Equation 1 for many solids has been found to be a constant of the order of 200. This includes such materials as the f c c metals, NaCl-structure alkali halides, and diamond cubic structure solids such as Ge, Si and diamond. The hardness of many such materials is

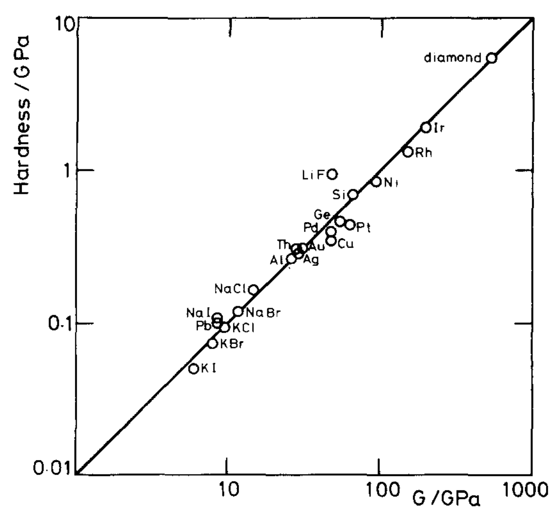

Figure 1 A comparison of the minimum hardness of many f c c metals, NaCl-structure alkali halides, and diamond cubic solids with their respective isotropic shear modulus, G.

compared with their shear moduli in Fig. 1. The value of the parameter A determined from the solid line drawn through these data using Equation 1 is in agreement with observed hardening rates in pure compression experiments.

A word must be said about the hardness values chosen for the preparation of Figure 1. For the f c c metals, the data of Fig. 1 represent the minimum room temperature values reported from various sources for high purity material. For these solids, room temperature is less than about one half the melting point. At temperatures above about one-half the melting point for f c c metals, diffusional creep makes a significant contribution to the material's plasticity. Atkins *et al* [8], while studying the time dependence of hardness in a number of these solids, found that the rate of change of the hardness with time was an increasing function of temperature which could be correlated with the activation energy of self-diffusion in the materials. Upon extrapolating the hardness back to zero time, however, they found that the short-time hardness approached a constant value independent of temperature. This would be the minimum glide hardness described by Equation 1.

In NaCl structures at room temperature, the yield strength and, therefore, the hardness is very sensitive to the defect concentration of the crystal. As the defect concentration is reduced the yield strength is reduced but the hardness approaches a minimum value [5]. This minimum value has previously been attributed to such mechanisms as the electrostatic faults that exist at the cores of {1 0 0} ⟨1 1 0⟩ dislocations [9] or the energy of formation of Schottky defects [10]. This paper, however, suggests that it is simply due to the work-hardening characteristics of the material.

For the diamond cubic materials Ge, Si and diamond, the room temperature hardness may not be a result of the normal plastic flow process as found, for example, in metals and alkali halides, but a critical pressure (or stress) dependent mechanism thought to be either the pressure dependent, semiconductor-to-metal phase transformation or athermal flow over the extremely strong Peierls barriers of these solids. Gerk [11] has shown that as the temperature is increased above about half the melting point, however, the hardness starts to rapidly decrease and becomes strongly time dependent. The hardness in this range is controlled by the low mobility growth of a plastic zone surrounding the indentation. As the temperature is further increased, dislocation mobility becomes high, the yield strength becomes very low, and the hardness approaches the constant, relatively time independent value which was used in the preparation of Fig. 1. These values are those measured at 800, 1000 and 1500° C respectively for Ge, Si and diamond. Diffusional creep is present at these temperatures as reflected in the minor time dependence of the hardness present; however, it is not thought to be a large factor in determining the hardness.

In view of the complicated nature of both the indentation process and second stage work-hardening, the degree of scatter in Fig. 1 is considered well within acceptable limits. Inaccuracies in the measured hardness values of the wide variety of materials used in Fig. 1 and the neglect of both plastic and elastic anisotropy by the use of a constant A and the average shear modulus will most likely account for the major portion of the scatter. The effect of anisotropy in the work-hardening coefficient can, in fact, be illustrated with the help of the alkali halides data in which the minimum hardness values are thought to be most accurately known. Second stage work-hardening in the NaCl-structure solids is thought to assume a much simpler macroscopic form than is the case for, say, the f c c metals. The dominant slip system in these materials is the ⟨1 1 0⟩ $\{1\,\bar{1}\,0\}$ which possesses only two independent slip systems. During the general deformation which occurs under an indentor, three more independent slip systems would be required. These are supplied by the ⟨1 1 0⟩ {0 0 1} system. The presence of ⟨1 1 0⟩ {0 0 1} slip during second stage work-hardening is well known and has been confirmed both by electron microscopy and slip-trace techniques for many of the NaCl alkali halides [12]. Second stage work-hardening may then be expected to reflect the long range elastic interaction between these systems. If one considers the long range elastic glide interaction forces between, say, a $[1\,1\,\bar{0}]$ $(1\,\bar{1}\,0)$ screw dislocation and ⟨1 1 0⟩ {0 0 1} type dislocation, most of the forces are proportional to an average elastic constant given by

$$G' = C_{44} \quad \frac{[(C_{11} - C_{12})/2C_{44}]}{1 + [(C_{11} - C_{12})/2C_{44}]}$$

This is the modulus which should be used in considering the alkali halides' minimum hardness rather than the isotropic shear modulus (Fig. 2). It is interesting to note that the LiF point which

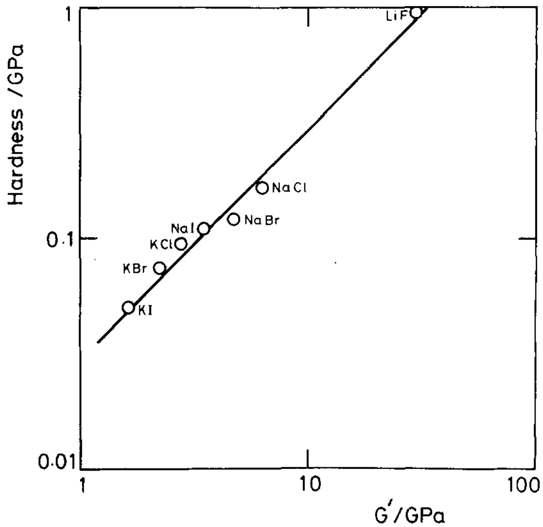

Figure 2 A comparison of the minimum hardness of NaCl-structure alkali halides with the average elastic coefficient, G', described in text.

was over a factor of two greater than the straight line value of Fig. 1 now agrees well with the rest of the alkali halide data of Fig. 2.

4. Conclusions

The assumption that the indentation hardness is three times the yield stress is valid only for a perfectly plastic solid. If the yield stress is comparable to the elastic modulus, the ratio can be much less than three. If the material work hardens appreciably, the ratio can be much greater than three.

In materials which work-harden the hardness approaches a minimum value when diffusional creep is not significant. This minimum value can be predicted by replacing the elastic constants of the elastic solution for wedges or cones by the rate of work-hardening. In many solids, including f c c metals, NaCl alkali halides, and the diamond cubic crystals, the rate of second stage work-hardening can be described by $\theta_{II} = G/A$ where A is found to be roughly constant. The minimum hardness of these solids, therefore, when plotted against their shear modulus, falls about the same line.

The amount of scatter in such a comparison can be greatly reduced when the details of work-hardening are considered. For example, for the NaCl-structure alkali halides, the rate of second stage work-hardening is not proportional to the isotropic shear modulus but to an appropriate average of the anisotropic moduli representing the glide interaction forces between $\langle 1\,1\,0 \rangle \{1\,\bar{1}\,0\}$ and $\langle 1\,1\,0 \rangle \{0\,0\,1\}$ dislocations.

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